Mathematical model and computing algorithm of thermal conduction during oil pumping process

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Abstract

Improving the methods of calculation modes of pipeline services allows to provide uninterrupted oil pumping process and increase its efficiency. Objective of the computing oriented to increasing work efficiency of current "hot" underground pipeline. The article considers several tasks of computer and mathematical modeling. There are: obtain a difference scheme for the equation of thermal variations, obtain an algorithm for calculating the equation using the double-sweep method, make a graph of temperature changes and develop program of mathematical model on JavaScript language.

Keywords: computer and mathematical modelling, double-sweep method, temperature changes.

1 Introduction

The technological process analysis of oil transportation in pipeline shows that the main problem in oil transportation is the changes of temperature requirements in oil pumping process that affected by different factors. The changes of temperature requirements in pipeline lead to the waxing problem and complete half of oil pipeline. Restarting of oil pipeline accompanied with many difficulties. By this reason, the development computer and mathematical model of heat condition of high-viscosity and thickening oil is the highly relevant objective.

2 Mathematical model

Solution of thermal transfer equation (Formula 1) in oil pumping process should be considered through the length of pipeline:

$$\left\{\pi R^2 k \frac{d^2T}{d\bar{x}^2} + \pi R^2 \rho C_p u \frac{dT}{d\bar{x}} - 2\pi R h_C\big(T - T_1(x)\big) = 0, \quad (1)\right.$$

where R – radius of circle, k – heat-conduction coefficient of oil, ρ – oil density, C_p – heat capacity of oil at constant pressure, u – oil pumping velocity, h_C – heat-exchange coefficient of pipe, $T_1(x)$ – function of temperature changes, T_0 – temperature on left side of border line, initial temperature ($T(0) = T_0$), T_L – temperature on right side of border line, final temperature ($T(l) = T_L$).

Take as a basis the scheme of pipeline between two pumping-heating stations (PHS). The distance between them is from 0 to l (Figure 1). At each of stations, will measured variety characteristics of high-viscosity oil.

Equations that obtained above necessary equate to a non-homogeneous differential formula of the second order (the Dirichlet boundary value problem). For simplifying the equation was executed following substituting:

$$\begin{cases} \phi = \frac{T-T_0}{T_L}, x = \frac{\bar{x}}{R}, x \in (0,\alpha), \alpha = \frac{l}{R} \\ a = \frac{Ru}{\alpha}, b = \frac{Rh_C}{k}, \sigma = \frac{k}{\rho C_p} \end{cases}. \quad (2)$$

By using a, b, σ quantities as new variables, it was reduced equation 1 to following difference scheme 3:

$$\begin{cases} \frac{y_{i+1}-2y_i+y_{i-1}}{(\Delta x)^2} + a\sigma \frac{y_{i+1}-2y_i}{\Delta x} + a(1-\sigma)\frac{y_i - y_{i-1}}{\Delta x} \\ -2by_i + 2b\frac{T_1(x_i)-T_0}{T_L} = 0 \end{cases}, \quad (3)$$

where $i = 1, 2, \dots, N-1$ and $y_0 = 0$, $y_N = 1 - \frac{T_0}{T_L}$.

Variables of oil temperature changes $T_1(x)$ along the length of pipeline conform with formula 4:

$$T_1(x) = \begin{cases} \left(\frac{x}{\frac{\alpha}{2}}\right)^2 + B_1\frac{x}{\frac{\alpha}{2}} + C_1, 0 < x < \frac{\alpha}{2} \\ -\left(\frac{x}{\alpha}\right)^2 + B_2\frac{x}{\alpha} + C_2, \frac{\alpha}{2} < x < \alpha \end{cases}. \quad (4)$$

There are boundary conditions, which align with a distinguishing exploitation characteristics of "hot" oil pipelines: $1 + B_1 + C_1 = -\frac{1}{4} + \frac{1}{2}B_2 + C_2$

1. At the input of pipeline oil temperature is defined and equals T_0: $T_1(0) = C_1 = T_0$
2. Intermediate value ($\frac{\alpha}{2}$) of oil temperature defined as 0.
3. At the output oil temperature is defined and equals T_L: $T_1(\alpha) = -1 + B_2 + C_2 = T_L$.

Based on the boundary conditions, it was determined the values B1, C1, B2, C2. The problem put by above listed conditions contains a unique solution and well defined.

3 Method of solving the problems of thermal processes and computing algorithm

It was solved approximately the equation 3, that included the equation of thermal variations (Formula 4) and border conditions, which hold for any randomly chosen interval of pipeline. In this regard was considered difference equation of the second order. It was obtainable following notation:

$$Ay_{i-1} - Cy_i + By_{i+1} + F = 0, \quad (8)$$

where $A, B \neq 0$, $y_0 = 0$, $y_N = 1 - \frac{T_0}{T_L}$, $i = 1, 2, \ldots$ N-1.

Using equation 3, it was determined necessary A, B, C, F parameters:

$$\begin{cases} A = 1 + a(1-\sigma)\Delta x; \quad B = 1 + a\sigma\Delta x \\ C = 2 + a\sigma\Delta x - a(1-\sigma)\Delta x + 2b(\Delta x)^2 \\ F = 2b\frac{T_1(x_i) - T_0}{T_L}(\Delta x)^2 \end{cases}. \quad (9)$$

The method of solving linear algebraic equations of the form $Ax = F$ accomplished by using double-sweep method, also known as Thomas algorithm. Algorithm based on method of sequential exclusion indeterminate variables. The main idea is to reduce the difference equation of second order to three difference equations of the first order, in general namely, non-linear. Suppose that recurrence equation is valid:

$$y_{i+1} = \alpha_i y_i + \beta_i. \quad (10)$$

Next step, transformation the equation 8 to form 10:

$$y_i = \frac{B}{C - A\alpha_i} y_{i-1} + \frac{F_i + A\beta_i}{C - B\alpha_i}. \quad (11)$$

This implies recurrence formula for α_{i+1} and β_{i+1}. Consequently, for determination y_N we get Cauchy problem i. e. equations of inverse double-sweep method:

$$y_N = \alpha_{N-1} y_{N-1} + \beta_{N-1}, \quad \alpha_{N-1} = 0, \quad \beta_{N-1} = 1 - \frac{T_0}{T_L}$$

Finally, put together the computing algorithm of right and left double-sweep method and write them in order of using:

$$\begin{cases} \alpha_{i+1} = \frac{B}{C - A\alpha_i}, \beta_{i+1} = \frac{F_i + A\beta_i}{C - A\alpha_i}, \quad \mathrm{i} = 1,2,\ldots,\mathrm{N}-1 \\ y_i = \alpha_{i+1} y_{i+1} + \beta_{i+1}, \mathrm{i} = \mathrm{N}-1, \mathrm{N}-2, \ldots, 1, 0 \end{cases}$$

4 Results

Implementation of mathematical model carried out from step by step, by accordance of solution concept. For implementation it was chose JavaScript language. Figure 1 demonstrate temperature curve along the length of the pipeline for T0 = 40 and TL = 60.

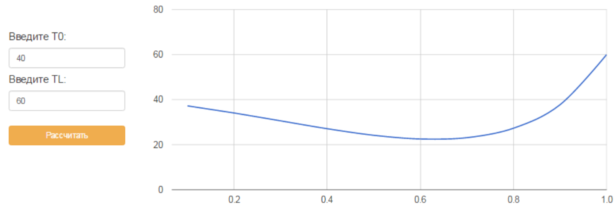

FIGURE 1 Temperature distribution along the length of pipe

5 Conclusion

The main task of oil transportation is providing the optimal temperature regime. During research was considered mathematical model and computing algorithm of thermal conduction during high-viscosity oil pumping process. As a result, the program of the task was implemented, including computing algorithm of temperature changes and graphic.

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