TRANSLATION THEOREM FOR DATABASE QUERIES
OVER A PARTIALLY ORDERED DOMAIN

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ABSTRACT

In relational model of databases, the state of a database is understood as a finite set of relations between
elements. Names of relations and its arities are fixed and refer to as the scheme of a database. The separate
information stored in the relations of the given scheme, refers to as a state of a database. Though
relational databases have been thought up for finite data sets, it is frequently convenient to assume that
there is an infinite domain – for example, the integer or rational numbers – so elements of the data get out
of this domain.

The signature of a relational structure \( L \) is non-empty set with the mapping assigning to each
relational symbol in \( L \) the relation of the same arity over this set. Let \( M \) be an infinite structure of
signature \( L \). Here we consider partially ordered structures. This means that \( L \) includes a binary relational
symbol \(<\) of which the interpretation in \( M \) satisfies to axioms of the partial order. We fix the scheme of
database SC and enter the following notations:

\[
L_0 = \{<\}, \quad L' = L_0 \cup SC, \quad L'' = L_0 \cup SC.
\]

A query of a database can be formally determined as a mapping, which is accepted by the state of a
database and it makes a new relation of fixed arity over \( M \). We consider two languages for querying.
Queries of the first language are formulas of signature \( L' \) – we name them by limited. Queries of the
second language are formulas of signature \( L'' \) – we name them by expanded.

Here we prove the translation theorem for database queries over a partially ordered domain.

Keywords: database query, partially ordered domain, weak o-minimality

1 GENERAL

The notion of o-minimality have been appeared more than twenty years ago [1] and proved its usefulness
and importance. Since that time many generalizations were appeared, name only some of them: weak o-
minimality [2], [3], circular minimality [4], weak circular minimality [5], o-stability [6], [7]. It is naturally
to try generalizing the notion of o-minimality on partially ordered structures that was originally done in
[8]. A structure of the form \( \langle M, =, <, \ldots \rangle \), where \( \langle M, < \rangle \) is a partially ordered set, is called a partially
ordered structure. In every partially ordered structure that is not linearly ordered the relation of non-
comparability of elements \( \emptyset \) is appeared, i.e. \( x \emptyset y := \neg(x = y) \land \neg(x < y) \land \neg(x > y) \).

Any family of pairwise incomparable elements of a partially ordered structure is called an antichain.
We say that a partially ordered structure has the width \( \leq \lambda \) if any its antichain contains no more than \( \lambda \)
elements. A set \( A \subseteq M \) is convex if for all \( a, b \in A \) and \( c \in M \) whenever \( a < c < b \) we have \( c \in A \). In
particular, points and intervals are convex sets. Obviously, antichains are also convex sets.

Our lecture concerns the notion of weak partial quasi-o-minimality originally studied by K.Zh.
Kudaibergenov in [8]. A weakly p.q.o.-minimal structure is a partially ordered structure \( M = \langle M, =, <, \ldots \rangle \)
such that any definable (with parameters) subset of \( M \) is a finite union of convex sets and \( \emptyset \)-definable
sets in \( M \). A theory \( T \) is weakly p.q.o.-minimal if every its model is weakly p.q.o.-minimal. Here we
present a criterion for connectedness of the set of realizations of every complete 1-type over \( M \) where \( M \)

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is a partially ordered structure of finite width. As corollary we receive reducibility of expanded queries to limited ones over a weakly $p.q.o.$-minimal domain having finite width.

We say that $k$-ary query $\Theta$ is locally generic over finite states if $\overline{a} \in \Theta$ if and only if $\varphi(\overline{a}) \in \Theta(\varphi(s))$ for any partial $<$-isomorphism $\varphi: X \to M$, where $X \subseteq M$, for any finite states over $X$ and for any $k$-tuple $\overline{a}$ in $X$.

We say that a complete theory $T$ has the Isolation Property if there is a cardinal $\lambda$ such that for any pseudo-finite set $A$ and for any element $\overline{a}$ of a model of the theory $T$ there exists $A \subseteq A$ such that $[A_0]$ $< \lambda$ and $tp(\overline{a}/A_0)$ isolates $tp(\overline{a}/A)$.

A set $A \subseteq M$, where $M$ is a partially ordered structure, is called connected if $A$ is convex and for all $a,b \in A$ - $(a \land b)$.

**Theorem 1.** Let $M$ be a partially ordered structure of finite width. Then $M$ is weakly $p.q.o.$-minimal iff the set of realizations of every complete $1$-type over $M$ is connected in any elementary extension of $M$.

**Theorem 2.** [9] Suppose that the complete theory of a structure $M$ has the Isolation Property. Then any expanded query being locally generic over finite states is equivalent to a limited query.

**Theorem 3.** Let $T$ be a weakly $p.q.o.$-minimal theory of finite width. Then $T$ has the Isolation Property.

The following corollary is the translation theorem for queries over a partial ordered domain:

**Corollary 4.** Let $T$ be a weakly $p.q.o.$-minimal theory of finite width. Then any expanded query being locally generic over finite states is equivalent to a limited query.

**References**


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