QUATTRO-20: advanced tool for estimation of the recurrent sequences

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Abstract

QUATTRO-20 as advanced tool for estimation of the recurrent sequences was created and tested. Several visualization methods such as final state diagram, distribution of Lyapunov exponent and CobWeb plot are included in package. Novel graphical technique of stability condition was presented and discussed.

Keywords: logistic map, discrete form of logistic equation, stability condition

1 Introduction

Stochastic processes are present in many fields of biology (population dynamics, host-parasite interactions), sociology (growth of population), economics (market activity), cryptography etc. [1] Modelling of stochastic processes is related to the usage of recurrence relations [2]. Evolution of physical/social/biological systems must be estimated in the framework of stability behaviour, and primary importance of such task is related to obtaining the various dynamical regimes of the certain system. One of the classical examples of recurrent relations is well-known logistic equation

$$x' = r \cdot x \cdot (1-x)\,. \tag{1}$$

Discrete form of logistic equation (1) allows to generate the sequence of terms as the function of the preceding terms:

$$x_{t+1} = r \cdot x_t \cdot (1-x_t)\,, \tag{2}$$

where x_t represents the current term and parameter $r>0$ represents the rate of population growth [2], [3]. Logistic map (2) represents parabolic function $F(x)$ at certain *r*.

Chaotic behaviour of dynamical system could be estimated using Lyapunov characteristic exponent $\lambda(r)$ which gives the rate of exponential divergence related to the initial condition:

$$\lambda(r) = \lim_{t\to\infty} \frac{1}{t} \sum_{k=0}^{t-1} \ln\left|F'(x)\,|_{x=x_k}\right|\,, \tag{3}$$

Distribution of Lyapunov exponent characters the chaotic dynamics as well as various forms of stabilization or synchronization [4]. Positive values of Lyapunov exponent indicates chaotic behaviour of sequence according to sensitive dependence on initial x0. A negative value of it indicates absence of chaotic dependence on initial x0. Several classical visualization methods such as final state diagram, distribution of Lyapunov exponent and CobWeb plot allow estimation of different aspects of sequence dynamics.

For learning purposes, program package *QUATTRO-20* [5] as an advanced tool was created and tested for estimation of the quantity and quality of recurrent sequences. Discrete form of logistic equation (2) was used as a model sequence. Previously mentioned visualizations methods are included in package. In addition, final state diagram was re-examined due to stability condition.

2 Overview

Let's consider the function $F(x)$ according to Equation (1):

$$F(x) = r \cdot x \cdot (1-x)\,. \tag{4}$$

We are interested in existing the fixed points x^* when $x^* = F(x^*)$. Locally stable attractors at x^* could be found from following stability condition [3], [6]:

$$\left|F'(x)\,|_{x=x^*}\right| < 1\,. \tag{5}$$

Actually, this condition must be checked for $F(x)$, $F^2(x)$, $F^3(x)$, $F^4(x)$ where

$$F^n(x) = F(F^{n-1}(x)), n = 2,3,4. \tag{6}$$

Estimation has to be divided into two parts.

Firstly, analyse of behaviour of function $F'(x)$ for *x* in interval [0, 1], and for *r* in (0, 4] with step 0.01 is shown in Figure 1.

Secondly, analyse of behaviour of recurrent formula (2) for the certain x_0 and for *r* in (0, 4] with step 0.01, is shown in Figure 2.

Values of first derivative of different functions were plotted as maps in Figure 1: $F(x)$ (top left), $F^2(x)$ (bottom left), $F^3(x)$ (top right), $F^4(x)$ (bottom right)

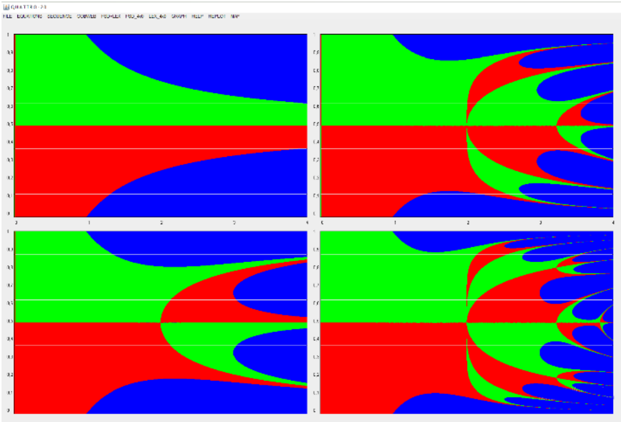

Figure 1 Map of first derivative of F(x) (top left), $F^{(2)}$(x) (bottom left), $F^{(3)}$(x) (top right), $F^{(4)}$(x) (bottom right) on parameters *r* at horizontal axis and *x* on vertical axis. Red area represents the derivative values in interval (0,1), green – (-1,0) and blue – all the rest.

where parameter r is on horizontal axis and x on vertical axis. Red area represents the interval where values of corresponding derivative are positive and less than 1, green area – are negative and greater than –1 and blue area – all the rest values. Figure 2 represents final state diagram (top left), distribution of Lyapunov exponent (bottom left), CobWeb plot (top right), and solution of equation $x = F(x)$.

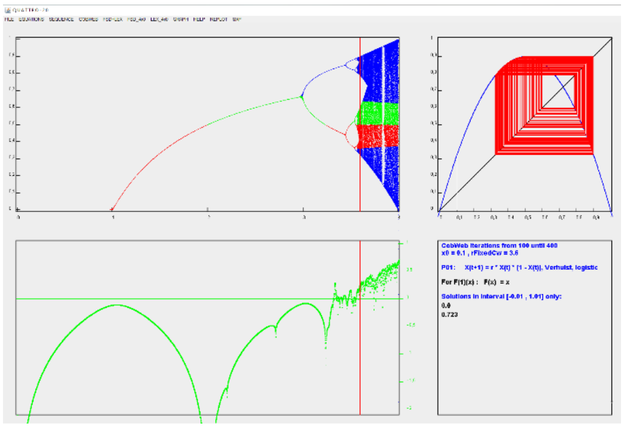

Figure 2 Different visualisations of F(x): final state diagram (top left), distribution of Lyapunov exponent (bottom left), CobWeb plot (top right), and solution of x=F(x) (bottom right). Parameter *r* at horizontal axis and *x* on vertical axis. Red, green and blue areas of final state diagram according to stability conditions as in Figure 1

(bottom right) for value *r*=3.6. Red vertical segment at final state diagram (at *r*=3.6) corresponds to the segment of the oscillations in the CobWeb plot. Iterations from requested interval [100, 400] are presented in red, another interval [1, 100) – in white (quite invisible). Construction and interpretation of CobWeb plot is widely described in [3].

3 Conclusion

Advanced tool for estimation of the recurrent sequences *QUATTRO-20* allows understanding the various aspects of chaotic system dynamics. Described package could be successfully used for other functions of recurrent sequences.

References

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