Algorithm for solving the inverse problem of kinematics by the position of the gripper of a manipulation robot using R-functions

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Abstract

The paper deals with the solution of the inverse problem of kinematics on the position of the gripper of a manipulation robot. The inverse problem of kinematics according to the position of a gripper is formulated as a problem for a conditional extremum, with a quadratic criterion and restrictions on the values of the generalized coordinates and R-functions describing the working space and subspaces of the manipulation robot. To solve this problem, a step-by-step algorithm has been developed.

Keywords: manipulation robot, kinematic system, workspace, inverse problem of kinematics in position

1 Introduction

Let the manipulation robot (MR) is an open-ended kinematic system [1], consisting of links and their sequentially connecting joints. MP articulations are kinematic pairs of the 5th class and can be given by the logical variables p_i

$$p_i = \begin{cases} 1, \text{if the articulation of the species is progressive displacement,} \\ 0, \text{if the joint is a kind of rotational movement.} \end{cases} \quad (1)$$

On the basis of the logical expression (1), it is possible to form a vector of joint types of the considered MR, $P(p_1, p_2, \cdots p_n)^T$. The amount of displacement is determined by the values of the generalized coordinates in degrees of mobility. MP, q_i *i=1,2,…,n*.

Control of MR in the space of coordinates associated with the base of MR is almost always associated with the need to solve the inverse problem of kinematics [1, 2]. To solve the inverse problem by position, it is necessary to specify the trajectory of movement, which is determined by the set of points $A_j(x_j, y_j, z_j)$, *j=1,2,…,m*, where m is the number of points approximating the trajectory.

It is necessary to minimize (maximize) the kinematic quality criterion of the species. [2]:

$$J = \sum_{j=1}^{m-1} \sum_{i=1}^{n} C_i \left(q_i^j - q_i^{j+1}\right)^2 \rightarrow \min\,(max), \quad (2)$$

under restrictions:

$$\forall A_j(x_j, y_j, z_j), j = \overline{1,m}: (D_1(x,y,z) \leq 0) L^R (D_2(x,y,z) \leq 0) L^R \cdots L^R (D_m(x,y,z) \leq 0) = 1, \quad (3)$$
$$q_i^{\text{н}} \leq q_i \leq q_i^{\text{в}}, \quad (4)$$

where $D_k(x, y, z) \leq 0$, (*k=1,2,…,m*) – inequality that specifies or approximates a part of the working space;
m – number of inequalities approximating workspace;
L^R – signs of logical operations R - conjunctions, R - disjunctions or R - negations.
C_i – the coefficient characterizing the dynamic performance of the drive i-the degree of mobility for a predetermined parameter, we will assume that $C_1 > C_2 > \cdots > C_n$;
q_i^j, q_i^{j+1} – the elements of Q^j and Q^{j+1}, respectively.
$q_i^{\text{н}}$, $q_i^{\text{в}}$ – the lower and upper values of the generalized coordinate of the *i* - the degree of mobility.

We describe the workspace MP L_0 as a logical expression (3), and the values of all the generalized coordinates $q_1, q_2, \cdots, q_n$ vary according to the degrees of mobility of the MP. Next, we describe the working subspace MP L_1 for a given value of the generalized coordinate q_1 of the first degree of mobility and the change of the generalized coordinates $q_2, q_3, \cdots, q_n$ of the second and following degrees of mobility. In a similar way, we obtain working subspaces $L_2, L_3, \cdots, L_{n-2}$.

The search step for solving the problem $\Delta q_1, \Delta q_2, \cdots, \Delta q_n$ in the degrees of mobility of the MP must satisfy the inequality $\Delta q_i < |q_i^{\text{в}} - q_i^{\text{н}}|$ and determined by the type of the degree of mobility p_i

$$\Delta q_i = \begin{cases} \Delta q_{\text{п}}, \; if \; p_i = 1, \\ \Delta q_{\text{в}}, \; if \; p_i = 0. \end{cases} \quad (5)$$

Let the initial position of the kinematic system MP is given by the vector $Q^j\left(q_1^j, q_2^j, \cdots q_n^j\right)^T$, the starting position of the grab position $A_j(x_j, y_j, z_j)$, and the required positioning point $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$, we define the parameters of these vectors, depending on the logical variables p_i, specifying the type of junction *i* - that degree of mobility

$$\exists p_i = 1: S_j = Sign(|\bar{a}_j| - |\bar{a}_{j-1}|), \exists p_i = 0: S_j = Sign(Arg\bar{a}_j - Arg\bar{a}_{j-1}), \quad (6)$$

where $|\bar{a}_j| = \sqrt{x_j^2 + y_j^2 + z_j^2}$, $Arg\bar{a}_j = arcsin\frac{z_j}{\sqrt{y_j^2+z_j^2}}$.

With this in mind, the above solution of problem (2), taking into account constraints (3), (4) and expressions (5), (6), the algorithm for solving the inverse problem on the MP position can be represented in the following steps.

2 Decision

START.
Step 1. Input of initial values: coordinates of points approximating the trajectory of the gripper $A_j(x_j, y_j, z_j)$, *j=1,2,...,m,*, logical expressions $L_1, L_2, \cdots, L_{n-2}$, describing the working space and the subspaces MP, $\Delta q_1, \Delta q_2, \cdots, \Delta q_n$, the magnitude of the solution search steps, the initial position of the configuration of MP MP $Q^0(q_1^0, q_2^0, \cdots q_n^0)^T$, the initial position of the gripper at the point $A_0(x_0, y_0, z_0)$.
If the condition $\forall A_j(x_j, y_j, z_j), j = \overline{1, m}: L_1 = 1$, then go to step 3, otherwise it is concluded that the task is unsolvable and go to the END.
Step 3. *j=1*.
Step 4. *i=1*.
Step 5. For a given value of q_i^j, we determine the type of logical expression L_i^{j-1}.
Step 6. If the coordinates of the point $A_j(x_j, y_j, z_j)$ do not satisfy the logical condition $L_i^{j-1} = 1$, then determine the sign according to the logical expression (6) and go to step 7, otherwise go to step 8.
Step 7. $q_i^j = q_i^{j-1} + S_j \cdot \Delta q_i$, go to step 6.
Step 8. *i=i+1*.
Step 9. If $i \leq n - 2$, then go to step 5, otherwise assign j = j + 1 and go to step 10.
Step 10. To determine analytically the values of the generalized coordinates are q_{n-1}^j, q_n^j.
Step 11. If j≤m, then go to step 4, otherwise go to step 12.
Step 12. Output matrix values $Q(q_i^j)$.
END.

3 Conclusion

As can be seen from the above algorithm, at each step a single value of the generalized coordinate is determined from the condition of the coverage of a given work subspace of a given point of the motion path. For the last 2 degrees of mobility, the inverse problem is solved analytically by position.

References

[1] Zenkevich S P, Yushchenko L S 2000 *Control robots. The basics of manipulating robots* M.: MSTU. N.E. Bauman 400 p
[2] 1984 *Dynamics of control of robots* Edited by E.I. Yurevich M.: Science p. 336
[3] Rvachev V L 1982 *The theory of R-functions and some of its applications* Kiev: Naukova Dumka, 530 p.